PLANCKS 2018

Physics League Across Numerous Countries for Kick-ass Students

Zagreb, Croatia

The International Physics Team Competition

Italy – Portugal - Spain Preliminaries

23/02/2018

Exercises

Introduction

Dear contestants,

In front of you, you have the exercises of the national preliminary of PLANCKS 2018. You are about to compete for a place to represent your country at the international competition in Zagreb, Croatia! These problems are being simultaneously solved by physics students from Italy, Portugal and Spain!

We hope you enjoy the competition! Before you start the competition, a few remarks must be made.

- Teams can consist of 3 to 4 students
- The language used in the preliminary and international competition is English
- The contest consists of 6 exercises each worth 20 points. Subdivision of points are indicated in the exercises.
- **All exercises must be handed in separately**
- You can use a dictionary: English to Portuguese
- You can use a non-scientific calculator
- The use of hardware (including phones, tablets etc.) is not approved, with exceptions of simple watches and medical equipment
- No books or other sources, except for this exercise booklet and a dictionary are to be consulted during the competition
- The organisation has the right to disqualify teams for misbehaviour or breaking the rules
- In situations to which no rule applies, the organisation decides

We wish you all the very best at the competition. May the best physics student teams represent their countries at PLANCKS 201!

Italy – Portugal – Spain PLANCKS Preliminaries Organisers

Contents

I. Thermodynamics

From general principles we know that the efficiency of a reversible Carnot cycle is always the same. Let us see how this work for real gases.

Let us start from Van der Waals equation

$$p(T,V) = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

where *a*, *b* are constant, *p* is the pressure, *T* the absolute temperature and *V* the volume.

Let us consider the Carnot cycle in *VT* plane whose corners are 1234. The upper left corner 1 has coordinates (V_1, T_1) and it is the starting point with minimal volume and maximal temperature. The lower right corner 3 has coordinates $(V_3, T_3 = T_2)$ and it is the point with maximum volume and lowest temperature. The paths 12 and 34 are adiabatic while 23 and 41 are isothermal.

Answer the following points.

a) Is the above cycle an engine or a heat pump?
b) Draw the cycle in *ST* plane.
c) The most general form for $E(T,V)$ compatible with the first principle of thermodynamics (Start from $dS = \frac{1}{T} dE + \frac{p(T,V)}{T} dV + \frac{\mu(T,V)}{T} dn$, consider $E=E(T,V)$, expand dE, require dS to be an exact differential, usescaling.)
d) The expressions for $\mu(T,V)$
e) The expressions for $S(T,V)$
f) For the adiabatic 12 find $V_2=V_2(T_1,T_2, V_1)$ and exchanged heat Q_{12}.
g) For the isothermal 23 find the adsorbed heat Q_{23}.

h) Using the previous results compute $Q_{released}/Q_{absorbed}$ (the heat is positive when absorbed and negative when released, i.e. $Q_{released} < 0$) and show it is equal to $-T_1/T_2$;

i) Is there any relation between the work done along the two adiabatics when $a \neq 0$? And when $a = 0$?

II. The Classical Zeeman Effect

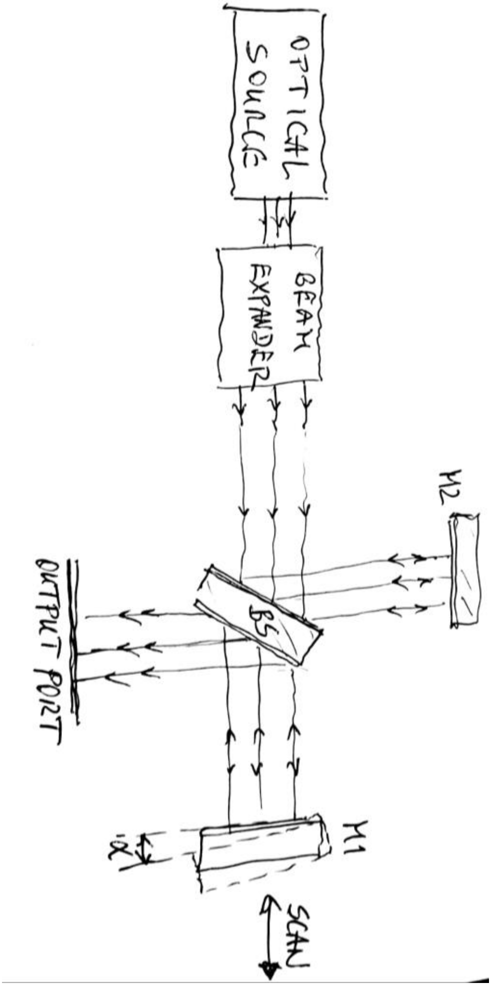
In 1897, long before the structure of atoms was understood, the Dutch physicist Pieter Zeeman published a series of papers in the Philosophical Magazine and Nature journals on “The effect of Magnetisation on the Nature of Light Emitted by a Substance”. In particular, he showed the splitting of the atomic spectral lines under the influence of an external magnetic field. This splitting is well understood in quantum theory but can also be addressed classically. This is the goal of the present problem.

A particle of mass m and charge –e, attracted by the Coulomb Field $V(r) = (Q/4\pi\epsilon_0 r)$ of a positive ion, is also subjected to the influence of a time dependent, homogeneous, magnetic field B(t). In this problem we will consider that the motion is non- relativistic and that $B(t)$ is perpendicular to the plane of the trajectory.

a) Find the electric field and the radial and tangential forces acting on the particle.
b) Prove that the quantity $M = L - (1/2)eB(t)r^2$, where $\boldsymbol{L} = m\boldsymbol{r} \times \dot{\boldsymbol{r}}$ is the particle angular momentum, is a constant of motion.
c) Consider for simplicity circular trajectories in what follows. The applied magnetic field is increasing in time from $B = 0$ initially. Find the value of *M* in terms of *α*, *m* and the initial radius r_0.
d) It is written in some books that the *B* field does not change the orbit radius. Show that this is not true and find the value of $\delta r = r_0 - r$ to second order in *B*.
e) Finally, using the previous results, show that the Zeeman formula $\omega - \omega_0 = eB/2m$ is only valid as a first order approximation (ωo is the angular velocity for $B = 0$).

III. Optics

The figure shows the scheme of an optical Michelson interferometer setup, which includes an optical source and a beam expander.



1. Optical source

Assume that the optical source is an external cavity He-Ne gas laser operating on the spectral line centred at 632.8 nm. The He-Ne discharge tube has Brewster windows, and the external reflectors constitute a cavity with length *L*. Pumping of the laser is sufficient to achieve emission on several longitudinal modes over that Ne spectral line. But in order to obtain single longitudinal mode emission with high spectral purity, a Fabry-Pérot étalon can be introduced in the laser cavity with adequate adjustment.

a) Assuming that the étalon is not yet inserted in the cavity, determine the minimum bandwidth of a photodetection system, directly placed at the laser output beam, capable of detecting emission on several cavity longitudinal modes.
b) Explain why linearly polarized and single longitudinal mode emission can be achieved with the étalon inserted in the cavity.

2. Beam expander

The laser emits a fundamental gaussian beam (*TEMoo*) with spot size w_o and divergence $\Delta\theta \approx \frac{\lambda}{\pi w_0}$, which is converted into a wider beam using a telescope expander (expansion factor *M*) built with two converging lenses (focal lengths $f_{1,2}$). A pinhole can also be inserted in the telescope to provide spatial mode filtering (i.e. to provide an output beam free of "spatial noise", as close as possible to a gaussian beam).

a) Define the beam expander configuration, determine the output beam divergence, and specify a suitable placement and diameter of the pinhole.

3. Michelson interferometer

The beam splitter (BS) is a parallel plate of silica (refractive index $n_{BS}(\lambda)$), thickness t_{BS}, incidence at 45º) with a dielectric multilayer coating on the front surface (amplitude splitting 50:50) and an antireflection coating on the back surface. The plane mirrors ($M_{1,2}$) are identical high reflectors; the scan arm mirror M_1 can be slightly tilted by an angle α relative to perfect alignment; the reference arm mirror M_2 is fixed and perfectly aligned. The interferometer arms are in air.

a) Assuming single frequency (ν) light and tilt angle α of M_1, determine the period Λ of the fringe pattern at the output port of the interferometer.

b) If a single element photodetector is placed at the output port plane (square sensitive area of size $D=\Lambda/4$), evaluate the contrast ($V = \frac{Smax-Smin}{Smax+Smin}$) of the output signal $S(t)$ due to mirror M_1 scanning.

IV. Cosmology

Friedman equations without General Relativity

A full and coherent description of the equations governing the expansion of the Universe (the Friedman Equations) requires General Relativity. However, it is possible to obtain their expression is a simplified way by using concepts of Newtonian Mechanics:

a) Consider a spherical portion of a homogenous distribution of mass with constant mass density ρ (mass density= mass per unit volume), that is uniformly expanding (see the figure below). Consider a point of mass *m* (call it "particle") at a distance $\boldsymbol{r}$ from a "center". By using Newtonian gravity and energy conservation, show that the equation that governs the motion of the particle is the following:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

where $kc^2 = -2E/mx^2$ (c is the velocity of light in vacuum), G is the Newton's constant, E is the total energy of the particle and the dot denotes derivative with respect to time, and where we have defined $\boldsymbol{r} = a(t)\boldsymbol{x}$. This definition means the following: since the medium is homogeneous, we can ascribe to the position of the mass "fixed" coordinates $\boldsymbol{x}$ (called comoving coordinates, they do not change in time) while the distance between the "centre" and the mass (the physical distance) changes in time, and evolves with *a(t).* The quantity *a(t)* is called the scale factor: it depends only on time since the space is homogenous, and its equation of motion is the above equation.

Notice that since the space is homogenous, there is not preferred position for the "centre": the same argument can be applied to any point is space. Think of an elastic blanket that can be stretched: any two points increase their distance in time according to the evolution of the scale factor *a(t).* This equation is the first Friedman Equation.

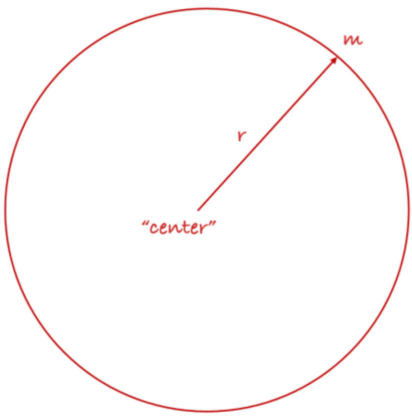
b) By using the first law of thermodynamics for the spherical portion of the fluid, show that the equation (the fluid equation) that governs the evolution of the matter density is (assume that expansion is a reversible adiabatic process):

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

c) By performing the time-derivative of Eq. from a) and using Eq. from b), show that the equation for the acceleration of the scale factor is:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

This is the second Friedman Equation. Only in General Relativity the term proportional to *k* can be properly understood in terms of the curvature of spacetime, but the equations are the ones here derived.



V. Quantum Mechanics

One of the most fundamental physics equation is the continuity equation stating that the change with time of the amount of a physical magnitude in a given volume equals the flux of an associated vector through the surface limiting the volume.

If the physical magnitude is the energy of a quantum state given by the Schrödinger equation when the potential is not time dependent, find the expression of the energy flux vector that fulfills the continuity equation.

VI. Solid State Physics

The atomic oscillations in solids are frequently studied using a simple model in which the chemical bonding behaves as an ideal spring linking atoms.

This model can be applied to estimate elastic constants of solids. In the following, assume an elemental metal, whose crystal structure is cubic simple. The atoms occupy the vertices of the cubes and they are linked to each next neighbour by a elastic spring of elastic constant k. Let be a the equilibrium distance between next neighbour atoms.

A cubic piece of this metal has edge L and tensile forces are applied perpendicularly to the faces of the cube.

a) Explain in which conditions the approach is valid.
b) Show that this simple model predicts a relative variation of sample length in the direction of applied force given by Hooke's law:

$$\frac{F}{L^2} = Y\,\frac{\Delta L}{L}$$

with Y the Young modulus. Calculate an expression of the Young modulus as a function of the parameters of the problem.

Partners