PLANCKS 20
preliminaries

Italy
14/02/20

EXERCISES

PLANCKS 2020
LONDON

{iaps}

AISF
associazione italiana studenti di fisica

Physics League Across Numerous Countries for Kick-ass Students

Italy Preliminaries

`Exercises`

14nd February 2020

Contents

Introduction

Dear contestants,
It is a pleasure to welcome you into the PLANCKS 2020 Italian Preliminaries! This booklet contains the exercises you will have to cope with in order to get access to the PLANCKS 2020 Finals in London, UK. You will face six different problems and are required to follow the instructions given in the next paragraphs.
Have fun and good luck!

1 Instructions

1.1 How it works

- The contest consists of 6 exercises each worth 20 points. Subdivision of points are indicated in the exercises.
- The test duration is for a total time of 2 hours and 30 minutes.
- When a problem is unclear, a participant can ask, via the crew, for a clarification from the organizing committee. The committee will respond to this request.
  If this response is relevant to all teams, we will provide this information to the other teams.
- In situations to which no rule applies, the organization decides.
- The organization has the right to disqualify teams for misbehaviour or breaking the rules.

1.2 What do you need

- You are only allowed to use a Italian/English dictionary and a non scientific calculator. **The use of hardware (including phones, tablets etc.) is not approved, with exceptions of simple watches and medical equipment**.
- No books or other sources, except for this exercise booklet and a dictionary are to be consulted during the competition.

1.3 What you are required to do

- The language used in the preliminary and international competition is English.
- Solutions **and** procedures must be included in the answer paper.
- All exercises must be handed in separately. **Please use a separate sheet for each problem.**
- Respect all the given instructions.
- Enjoy and have a great physics time!

2 Problems

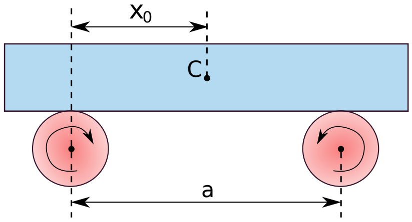
2.1 Mechanics

Two cylinders and a beam

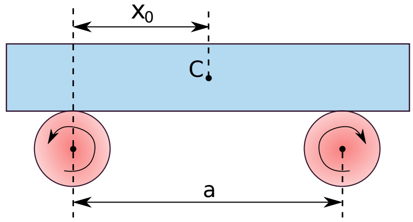
A rigid bar with mass M lays on two cylinders, each of them revolves around a central fixed axis and the two axes are distant a. The friction coefficient between the bar and the cylinders is μ. At the time $t = 0$ the bar stays such that:

$$x(0) = x_0; \qquad \dot{x}(0) = 0$$

where $x(t)$ is the position of the bar center with respect to the left-hand cylinder axis.



(a) Configuration for Question 1



(b) Configuration for Question 2

If the two cylinder revolve in opposite directions, compute:

1. $x(t)$ for the configuration in Fig.2.1a **[10 points]**
2. $x(t)$ for the configuration in Fig.2.1b **[10 points]**

2.2 Thermodynamics

Thermodynamic properties of a rod

We want to study some properties of an elastic rod, with the help of thermodynamics. In order to do that we will use a simple phenomenological model where the rod's entropy is written as a function of the two extensive thermodynamical parameter of interest, the energy $\mathcal{U}$ and the length ℓ_0, as:

$$S(\mathcal{U}, \ell) = k\,\ell_0\, log\left(\frac{\mathcal{U}}{\mathcal{U}^*}\right) - \frac{\beta}{2}(\ell - \ell_0)^2 \tag{2.1}$$

where k, $\mathcal{U}^*$, k are constants with appropriate dimensions. Eq.2.1 gives a complete description of the rod from a thermodynamical point of view.
In the following we will explore some properties of this model, taking always $\ell > \ell_0$ while the tension on the rod will be indicated with a τ.

1. Write the infinitesimal expression of the first principle. Then,using Eq.2.1, derive the equations of state $\mathcal{U} = k\,\ell_0\, T$ and $\tau = \beta\, T(\ell - \ell_0)$. **[1 points]**

2. Find the heat capacity at constant length and the heat capacity at constant tension of the rod. **[3 points]**

3. Write the expression of the work that must be done on the rod to slowly change its length from ℓ_0 to ℓ_f, supposing that the rod is in thermal contact with an heat bath at a fixed temperature T. **[2 points]**

4. We slowly change the tension of the rod from 0 to τ_f. Supposing that the rod can't exchange heat with the environment and that its initial temperature is T_0, find the final relative length variation:

   $$\epsilon = \frac{\ell - \ell_0}{\ell_0}$$

   in a small τ_f approximation. Give meaning to this approximation. **[4 points]**

5. Now we change again the tension of the rod from 0 to τ_f , but varying instantaneously the external force which acts on it. The rod still can't exchange heat with the environment and its initial temperature is again T_0. What is the variation of entropy? Find the final relative length variation in a small τ_f approximation and compare with the result of the previous exercise. **[5 points]**

6. Two identical rods described by the model are constrained at a fixed length $T_1 > T_2$. They have initially two different temperatures T_1 and T_2. Evaluate the maximum amount of work that can be obtained from the system without changing the length of the rods but using them as hot and cold sources for a thermal machine.**[5 points]**

Electron in 1D harmonic potential

An electron in a one-dimensional harmonic potential is described by the hamiltonian

$$H_0 = \omega S_z + \hbar\omega(a^\dagger a + 1/2)\,, \tag{2.2}$$

where S_z is the z-component of the spin operator, and a and $a^\dagger$ are the usual ladder operators.

1 Deduce the eigenstates of H_0, the corresponding eigenvalues, and their degeneracy. **[6 points]**

2 At time $t = 0$ the electron is in a harmonic eigenstate, with spin up along x. Find out the probability $\mathcal{P}(t)$ to measure, at times $t > 0$, spin down along a generic direction $\hat{n}$ in the $x-z$ plane, at angle θ from the z axis. Find the values of θ and t for which $\mathcal{P}(t) = 1$. Discuss the dependence of this result upon the harmonic state the electron is in. **[8 points]**

Add to H_0 a perturbation

$$H_1 = \frac{u}{2}(aS_+ + a^\dagger S_-)\,, \tag{2.3}$$

where $S_\pm = S_x \pm iS_y$, and $u = 2\pi\omega$.

3 Deduce the eigenstates of $H_0 + H_1$, and the corresponding eigenvalues. **[6 points]**

2.4 Particle Physics

B^0-$\bar{B}^0$ oscillation and decay

At the interaction point of high-energy colliders, unstable neutral *B-mesons*, can be created as particle-antiparticle (B^0-$\bar{B}^0$) pairs. Like other unstable particles, such mesons decay with an exponential probability distribution over time. However, B^0 and $\bar{B}^0$ have the peculiar property of "oscillation", meaning that they can spontaneously change into the respective antiparticle before decaying.
As such, the probability for the meson produced as B^0 to decay in a time interval $[t, t + dt]$ still as a B^0 is:

$$\mathcal{P}(B^0(t)|B^0(0)) = Ne^{-\frac{t}{\tau}}[1 + \cos(mc^2 t/\hbar)],$$

where N is a probability normalization constant, τ and m are parameters specific to the meson species, $\hbar = 6.582 \cdot 10^{-22}$ MeV $\cdot$ s is the reduced Planck constant and c is the speed of light.
The probability for the meson produced as B^0 to decay in a time interval $[t, t + dt]$ but having oscillated to a $\bar{B}^0$ is instead:

$$\mathcal{P}(\bar{B}^0(t)|B^0(0)) = Ne^{-\frac{t}{\tau}}[1 - \cos(mc^2 t/\hbar)].$$

Symmetric relationships hold for the meson produced as $\bar{B}^0$. Depending on the pair production mode, oscillations can be a) *coherent* (at a given time t_0 there is always a B^0 and a $\bar{B}^0$ in the event) or b) *incoherent* (oscillations of the two mesons occur independently).

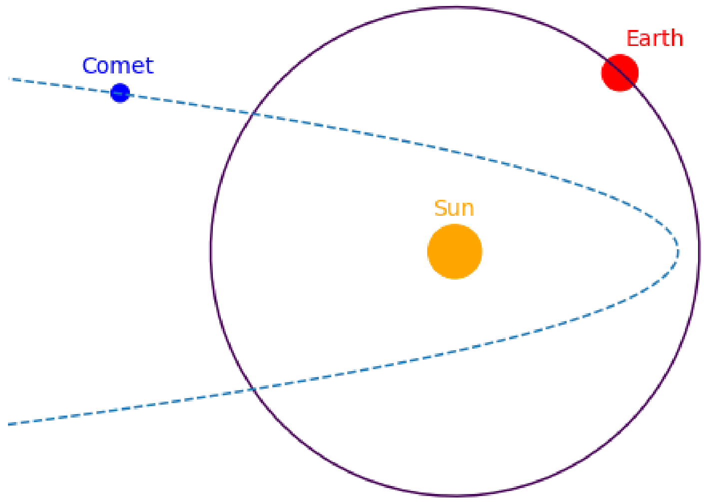
1. Show that the probability distribution for B^0 to decay in a time interval $[t, t + dt]$ is exponential and determine the constant N. **[4 points]**

2. Determine the total probability χ that the produced B^0 decays as a $\bar{B}^0$ **[6 points]**

3. Compute the value of χ for B^0_d mesons ($\tau_{Bd} = 1.52 \cdot 10^{-12}$ s and $m_{Bd} = 3.33 \cdot 10^{-10}$ MeV/c^2) and B^0_s mesons (τ_{Bs} and m_{Bs} are not measured precisely but it is known that $m_{Bs} c^2 \tau_{Bs} \gg \hbar$). **4 points]**

4. Compute the total probability to observe two B^0 decays or two $\bar{B}^0$ decays in the same event in case a) and b). **[6 points]**

Comet under gravitational field



Assume the Earth orbit to be perfectly circular and belong to the 2D space $\mathcal{P}$. Consider a comet C with mass m following a parabolic trajectory in the same plan $\mathcal{P}$. Compute the maximum amount of time that the comet spends inside the Earth orbit.
[20 points]

2.6 Solid State Physics

1D chain of ions with opposite charges

Consider a 1D chain of 2N ions having opposite charges $\pm q$ with R nearest neighbours distance. On top of the electrostatic interaction, consider also a repulsive interaction between nearest neighbours only $\propto \frac{A}{R^b}$ with A, b given constants of the lattice.

1. Derive an expression for the total energy of the 1D chain.**[10 points]**
2. Find the equilibrium nearest-neighbor distance R_0. **[3 points]**
3. Evaluate the lattice energy per ion pair.**[3 points]**
4. Evaluate the work needed to make the crystal shrunken to such an extent l its nearest-neighbor distance is reduced by a fraction η.**[4 points]**