PLANCKS 21
preliminaries

ITALY
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BOOKLET

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PLANCKS

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PrePlanks 2021

Associazione Italiana Studenti di Fisica

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1 Introduction

Dear contestants, It is a pleasure to welcome you into the PLANCKS 2021 Italian Preliminaries! This booklet contains the exercises you will have to cope with in order to get access to the PLANCKS 2021 Online Finals in Porto, Portugal. You will face five different problems and are required to follow the instructions given in the next paragraphs. Have fun and good luck!

2 Instructions

2.1 How it works

- The contest consists of 5 exercises each worth 20 points. Subdivision of points are indicated in the exercises.
- The test duration is for a total time of 2 hours and 30 minutes.
- When a problem is unclear, a participant can ask, via the crew, for a clarification from the organizing committee. The committee will respond to this request. If this response is relevant to all teams, we will provide this information to the other teams.
- In situations to which no rule applies, the organization decides.
- The organization has the right to disqualify teams for misbehaviour or breaking the rules.

2.2 What do you need

- You are only allowed to use a Italian/English dictionary and a non scientific calculator. **The use of hardware (including phones, tablet set c.) is not approved, with exceptions of interface and discussing via Google Meet, watch this booklet or medical equipment.**
- No books or other sources, except for this exercise booklet and a dictionary are to be consulted during the competition.

2.3 What you are required to do

- The language used in the preliminary and international competition is English.
- Solutions **and** procedures must be included in the answer paper.
- All exercises must be handed in separately. **Please use a separate sheet for each problem.**
- Respect all the given instructions.
- **Enjoy and have a great physics time!**

3 Problems

3.1 Mechanics

A particle with mass m moves on a plane, subjected to a constant force $\vec{F} = (F, 0, 0)$. Knowing that its initial velocity is $\vec{v_0} = (0, v0, 0)$ and that the frictional force is equal to F , find the terminal velocity of the particle. [**20 points**]

Alessandro Papa, University of Calabria.

3.2 Quantum Mechanics

Consider the Kepler-like Hamiltonian

$$H = \frac{\vec{p}^2}{2m} + \frac{\alpha}{r} \ (1)$$

with $r = \sqrt{x^2 + y^2 + z^2}$, $\vec{p} = (p_x, p_y, p_z)$, $\vec{p}^2 = p_x^2 + p_y^2 + p_z^2$ and α a real constant with the dimension of energy times length. Let $|\psi_E >$ be a vector in the Hilbert space $H = L^2(R)$ such that H$|\psi_E >$ = E $|\psi_E >$. The virial relation says that:

$$\text{E} = \text{-} < \frac{\vec{p}^2}{2m} >_{\psi_E} = \frac{1}{2} < \frac{\alpha}{r} >_{\psi_E}, \ (2)$$

Where $< O >_{\psi_E} = < \psi|O|\psi >$ is the mean values of the operator O : $H \to H$ with respect to the state vector $|\psi >$. The standard lore has equation (2) derived from:

$$0 = < [H, r \cdot \vec{p}] >_\psi = \text{-2i}\hbar \ (\text{E - } \frac{1}{2} < \frac{\alpha}{r} >_{\psi_E}), \ (3)$$

where $\vec{r}$ = (x,y,z) and $r\vec{p} = xp_x + yp_y + zp_z$ Technically speaking, however, the mean value of the commutator is in general well defined only when the eigenstate ψ_E belongs to the domain of the operator $\vec{r} \cdot \vec{p}$. In order to overcome this inconvenience, one introduces the one parameter family of unitary operators:

$$U_\lambda = exp(-i\lambda \frac{\vec{r} \cdot \vec{p} + \vec{p} \cdot \vec{r}}{2}), \lambda \in R. \ (4)$$

1. Compute the commutator $[H, r \cdot \vec{p}]$ writing $r \cdot \vec{p}$ in spherical polar coordinates and momenta, (r, θ, ϕ) and (p_r, p_θ, p_ϕ), whereby:

$$\vec{p}^2 = p_r^2 + \frac{L^2}{r^2}, \ (5)$$

with L the angular momentum operator. [**4 points**]

2. Explain why one needs $\frac{\vec{r}\cdot\vec{p}+\vec{p}\cdot\vec{r}}{2}$ in U_λ and not simply $\vec{r} \cdot \vec{p}$. [**3 points**]

3. Show that U_λ implements the dilatations:

$$U^\dagger{}_{\vec{r}} U_\lambda = e^\lambda \vec{r},$$
$$U^\dagger{}_{\vec{p}} U_\lambda = e^{-\lambda} \vec{p}$$

[**3 points**]

4. Show that:

$$< \psi_E| = (\frac{\vec{p}^2}{m}(e^{2\lambda} - 1) + (e^\lambda - 1)\frac{\alpha}{r})U_\lambda|\psi_E >= 0, \forall \lambda \in R \ (6)$$

[**3 points**]

5.Use (6) to prove the virial relation (2). [**2 points**]

6. Using (2), show that in the case of an attractive Kepler potential, the eigenvlalues of H must be negative, while if the potential is repulsive there are no proper eigenstates of H. [**5 points**]

F. Benatti, University of Trieste.

3.3 Thermodynamics

An idealized model of the Earth's atmosphere is bsed on the following assumptions:

- the Earth's surface temperature has a uniform value of $T_0 = 288$K;
- the atmosphere is a monatomic gas of a molecular weight 29;
- the system is spherically symmetric and the temperature and pressure vary only in the radial direction;
- the atmosphere is in a steady state and is stationary - there is no convection.

The thermal conductivity at sea level is $k_0 = 2.5$ x $10^{-2} W m^{-1} K^{-1}$ and the pressure at sea level is $P_0 = 10^{-5} P_a$. The radius of the Earth is $r_0 = 6.4$x10^{-3} km.

(a) Suppose that the height of the atmosphere is sufficiently small that one can consider just a vertical column of the air with the constant g, the acceleration due to gravity. Find the variation in pressure with altitude z for an isothermal atmosphere (in which the temperature is constant). [**4 points**]

(b) Interpreting the relaxation τ as the mean collision time, and assuming that the scattering cross-section σ is constant, how does τ depend on pressure and temperature? [**4 points**]

(c) Near the Earth's surface, the atmosphere actually has a temperature gradient dT/dz = -α_0, with $\alpha_0 \approx 6x10^{-3} Km^{-1}$. Modify the analysis of (a) to find the variations in pressure and temperature with altitude, using this boundary conditions. You should find that there is a maximum altitude z_{max} beyond which this model does not work. What is the value of z_{max}? [**4 points**]

(d) Why does the analysis n (c) go wrong? Estimate the altitude up o which the analysis might be reasonable. [**4 points**]

(e) Find the variation in pressure with altitude in a isothermal atmosphere, taking in to account that the Earth is (very nearly) spherical, and the variation in the gravitation force with distance with the center of Earth. You should find that a steady-state solution is possible only if the Earth is immersed in an infinite gaseous medium. What are the pressure and number of density of this medium a long way from the Earth? [**4 points**]

G. Gonnella, University of Bari.

4 Particle Physics

Antiproton Discovery

The antiproton was discovered in 1955 using the inelastic pp scattering of energetic protons on a copper target. In the following, consider a single target proton at rest, neglecting effects related to the copper nucleus and of the motion of the proton inside the nucleus.

- Write the minimal pp reaction compatible with baryon number conservation that can produce antiprotons, and determine the p beam energy threshold E^{th}_{beam} beam for the reaction. [**3 points**]

  For the following questions assume that the the beam total energy is set to E_beam = 7,5 GeV = 8 m_p, where m_p is the proton mass. [**4 points**]

- Compute the maximum energy $E^{*max}_{\bar{p}}$ and maximum 3-momentum $p^{*max}_{\bar{p}}$ of the anti-proton in the pp center of mass system (CMS). [Hint: you can assume the maximum energy configuration is obtained when all the other particles in the final state move as single particle against the antiproton.] [**4 points**]

- Compute the minimum and maximum energy $E_{\bar{p}}^{min}$, $E_{\bar{p}}^{max}$of the antiproton in the laboratory system (LABS). [**4 points**]
- Discrimination against the copious π^- production can be done using Time Of Flight (TOF) measurements. The negative particles are selected with a spectrometer around a momentum in the LABS pTOF $= 2m_p =$ 1,88 GeV. Compute the TOF difference between $\bar{p}$ and π^- over a distance L=12 m. [**5 points**]

Some notes:

- Energies, momenta and masses are all be expressed in energy units, e.g. GeV, GeV/c, Gev/c^2.
- Answers can also be expressed as multiple of the proton mass m_p.
- Masses: $m_p = m_{\bar{p}} = 0{:}938$ GeV/c^2; $m_{\pi^-} = 0{:}140$ GeV/c^2.

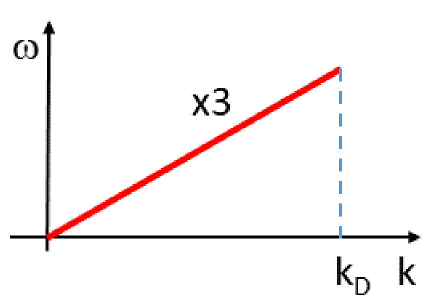
F.Forti, University of Pisa

4.1 Solid State Physics

Phonons in solids are obtained by quantizing lattice vibrations in the harmonic approximation and are described by the following Hamiltonian:

$$H = \sum_{\mathbf{k}s} \hbar\omega_{\mathbf{k}s} \left(\hat{n}_{\mathbf{k}s} + \frac{1}{2} \right), \tag{1}$$

where $\mathbf{k}$ is the wavevector, s is the branch index, $\omega_{\mathbf{k}s}$ are the frequencies of classical normal modes, and $\hat{n}_{\mathbf{k}s}$ is the phonon number operator. At temperature $T = 0$, the energy of the lattice ground state is given by the *zero-point energy* $E_0 = \frac{1}{2} \sum_{\mathbf{k}s} \hbar\omega_{\mathbf{k}s}$.



This term can be estimated in the Debye model, which assumes three acoustic phonon branches with linear dispersion $\omega_{\mathbf{k}s} = ck$ for $k \leq k_D$, where c is the speed of sound. The maximum wavevector k_D is called the *Debye wavevector* and is determined by the condition that the number of normal modes of vibration, $3\sum_{\mathbf{k}}$, equals the total number of degrees of freedom given by $3N_i$, where N_i is the number of ions.

1. Demonstrate that the Debye wavevector is given by $k_D = (6\pi^2 n_i)^{1/3}$, where $n_i = N_i/V$ is the ion density. [**5 points**]

2. Calculate k_D for aluminium, knowing its mass density $\rho_m = 2.7$ g/cm^3 and its atomic mass number $A = 26.98$. Assuming an average speed of sound $c = 3420$ m/s, calculate the Debye energy E_D and the Debye temperature T_D, defined by $E_D = \hbar c k_D = k_B T_D$. [**4 points**]

3. Derive an expression for the zero-point energy E_0 in the Debye model. [**3 points**]

4. Calculate E_0 , expressed in electron-volts (eV) per atom, for the case of aluminium. Does the zero-point energy *increase* or *reduce* the cohesive energy? Knowing that the measured cohesive energy of aluminium is 3.39 eV/atom, what is the ratio of phonon zero-point energy to cohesive energy? [**4 points**]

5. The zero-point energy can be interpreted as being due to quantum fluctuations of the ionic positions in the phonon ground state. Give an estimate of the root-mean-square amplitude of the ionic motion, $\Delta u = \sqrt{\langle u^2 \rangle}$, by equating the quantum kinetic energy to the Debye energy. Evaluate Δu for the case of aluminium. [**4 points**]

Useful constants: $\hbar = 1.05 \cdot 10^{-34}$ J·s, $k_B = 1.38 \cdot 10^{-23}$ J/K, proton mass $m_p = 1.67 \cdot 10^{-27}$ kg. A sum over wavevectors can be transformed into an integral as follows: $\sum_{\mathbf{k}} = \int \frac{V d^3\mathbf{k}}{(2\pi)^3}$.

L. Andreani, University of Pavia