PrePlanks 2022

Associazione Italiana Studenti di Fisica

March 4th, 2022

1 Introduction

Dear contestants, it is a pleasure to welcome you into the PLANCKS 2022 Italian Preliminaries! This booklet contains the exercises you will have to cope with in order to get access to the PLANCKS 2022 Finals in Munich, Germany. You will face four different problems and are required to follow the instructions given in the next paragraphs. Have fun and good luck!

2 Instructions

2.1 How it works

- The contest consists of 4 exercises each worth 20 points. Subdivision of points are indicated in the exercises.
- The test duration is for a total time of 2 hours and 30 minutes.
- When a problem is unclear, a participant can ask, via the crew, for a clarification from the organizing committee. The committee will respond to this request. If this response is relevant to all teams, we will provide this information to the other teams.
- In situations to which no rule applies, the organization decides.
- The organization has the right to disqualify teams for misbehaviour or breaking the rules.

2.2 What do you need

- You are only allowed to use a Italian/English dictionary and a non scientific calculator. **The use of hardware (including phones, tablet set c.) is not approved, with exceptions of interface and discussing via Google Meet, watch this booklet or medical equipment.**
- No books or other sources, except for this exercise booklet and a dictionary are to be consulted during the competition.

2.3 What you are required to do

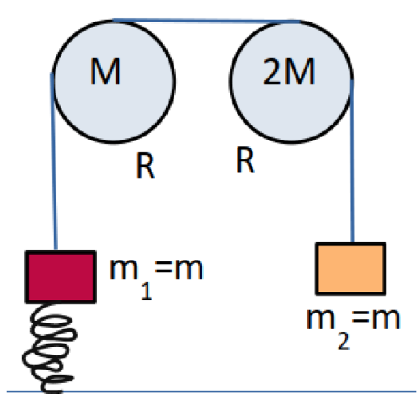
- The language used in the preliminary and international competition is English.
- Solutions **and** procedures must be included in the answer paper.
- All exercises must be handed in separately. **Please use a separate sheet for each problem.**
- Respect all the given instructions.
- **Enjoy and have a great physics time!**

3 Problems

3.1 Mechanics

A point-like mass $m_1 = m_2 = 2$kg is connected to a mass-less spring, with constant k, and to an ideal mass-less string. The string runs over two massive constrained pulleys with masses M = 10 kg and 2M, and radiuses R = 10 cm and 2R respectively, and is connected at the other end to another point-like mass $m_2 =$ m. If the mass m_1 is moved downwardly of $\Delta z = 3$cm with respect to the equilibrium, and then it is left free to move, the system oscillation period is T = 3.4 s.

- What is the spring constant k? **(5 points)**
- What is the m_1 velocity after $\Delta t = 1.0$ s from its release? **(5 points)**
- What is the m1 gravitational potential energy difference with respect to equilibrium, after $\Delta t = 1.0$ s from its release? **(10 points)**



Prof. Davide Basilico, University of Milan.

3.2 Thermodynamics

Consider a spherical soap bubble exposed to an external pressure $P_e = 10^5 Pa$. The pressure inside the bubble is $P_i = (1 + 10^{-4})P_e$ due to a surface tension $\tau = \frac{3}{100} P_a$ m.

- Determine the radius of equilibrium of the bubble, neglecting its thickness. **(10 points)**
- Justify this approximation qualitatively. **(10 points)**

Prof. L. Cremonesi, University of Milan.

3.3 Quantum Mechanics

The Hamiltonian of a one-dimensional harmonic oscillator of frequency ω and mass m is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 \,. \tag{1}$$

a)
Derive the expressions of the operators a and $a^\dagger$ such that $[a, a^\dagger] = 1$ and $\hat{H} = \hbar\omega(a^\dagger a + 1/2)$ **(7 points)**.

b)
The operator $\hat{N} = a^\dagger a$ has integer eigenvalues $n \geq 0$ and the operators a and $a^\dagger$, acting on a state with eigenvalue n, produce a state with eigenvalue $n-1$ and $n+1$, respectively, up to a normalization factor. Use the operators a and $a^\dagger$ to show that the mean value of the kinetic energy is equal to the mean value of the potential energy when the particle is in any eigenstate of $\hat{H}$ **(5 points)**.

c)
Find the temporal evolution of the operators a and $a^\dagger$ in the Heisenberg representation **(4 points)**.

d)
Determine the selection rules for transitions between states with different n induced by a perturbations whose spatial dependence is x^4 **(4 points)**.

Proff. F. Dalfovo and S.Giorgini, University of Trento.

3.4 Solid State Physics

Consider a two-dimensional (2D) crystal made out of identical atoms, as shown in the figure, where $a = 5\,\text{Å}$ and $b = 2.8\,\text{Å}$ are the edges of a primitive unit cell.

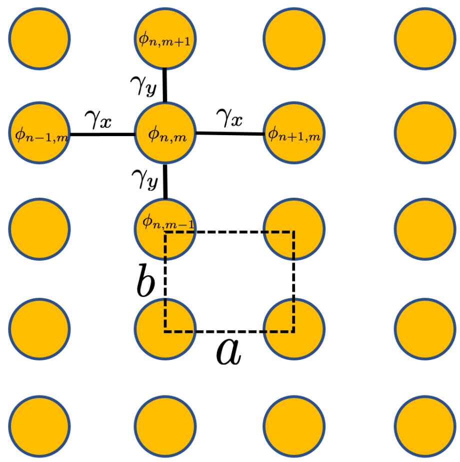


Figure 1: Lattice.

Under the usual approximation of independent electrons, the only non-zero matrix elements of the electronic Hamiltonian H_e are (see Fig. 1)

$$\langle \phi_{m,n}|H_e|\phi_{m\pm1,n}\rangle = -\gamma_x\,, \quad \langle \phi_{m,n}|H_e|\phi_{m,n\pm1}\rangle = -\gamma_y \qquad (m, n \in Z)$$

(and respective c.c.) with

$$\gamma_x = 0.2\,\text{eV}\,, \quad \gamma_y = 0.6\,\text{eV}\ .$$

Here, $\phi_{m,n}$ is an s-like orbital centered on the atom sitting at the lattice site indexed by (m, n). Also, assume that the energy offset is such that $\langle \phi_{m,n}|H_e|\phi_{m,n}\rangle = 0$ for any m, n. The overlaps between orbitals of different atoms can be neglected.

1. Determine the first Brillouin zone (FBZ) of the lattice. **(3 points)**

2. By applying the tight-binding method, calculate the dispersion law $E(\mathbf{k})$ of the s-like band arising from the overlap of all the ϕ_{mn}'s orbitals and the resulting bandwith. Plot the energy band along the (1,0) and (1,1) cuts of the FBZ. **(8 points)**

3. Assume that the density of conduction electrons is such that the Fermi energy lies at $E_F = -1.5\,\text{eV}$. Work out an approximated expression for the dispersion law of the conduction electrons and the corresponding Fermi surface (the latter being actually a contour line since the lattice is 2D). **(4 points)**

4. Use the result at (3) to compute the spatial density of conduction electrons as a function of E_F. **(5 points)**

Prof. Francesco Ciccarello, University of Palermo & NEST.