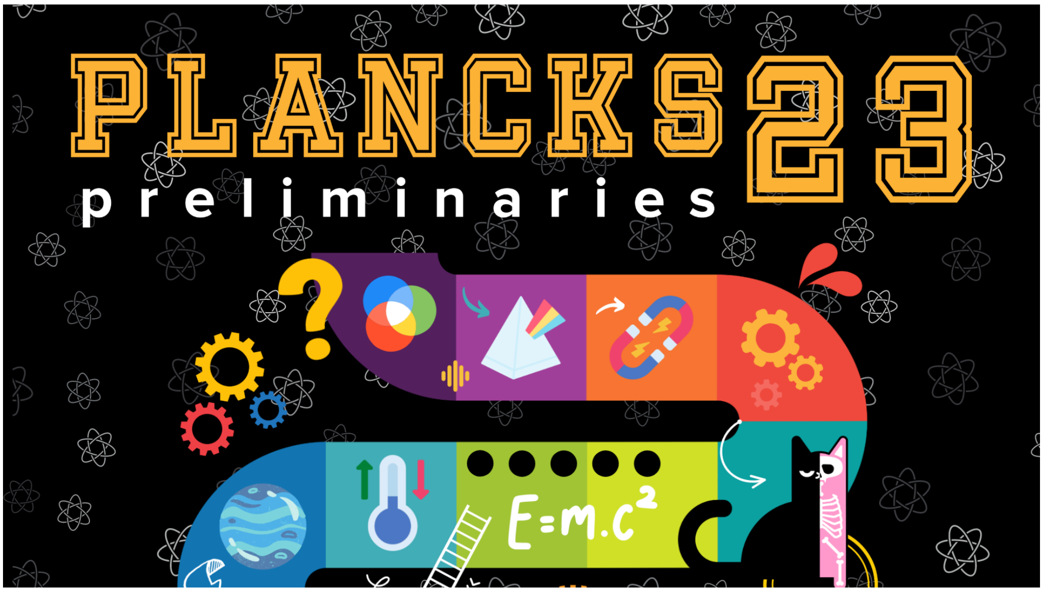
PrePlanks 2023

Associazione Italiana Studenti di Fisica

March 22th



Dear contestants, It is a pleasure to welcome you into the PLANCKS23 Italian Preliminaries! This booklet contains the exercises you will have to cope with in order to get access to the PLANCKS23 Finals in Milan, Italy. You will face five different problems and are required to follow the instructions given in the next paragraphs.
Have fun and good luck!

Instruction

How it works

- The contest consists of 5 exercises each worth 20 points. Subdivision of points are indicated in the exercises.
- The test duration is for a total time of 2 hours and 30 minutes.
- When a problem is unclear, a participant can ask, via the crew, for a clarification from the organizing committee. The committee will respond to this request. If this response is relevant to all teams, we will provide this information to the other teams.
- In situations to which no rule applies, the organization decides.
- The organization has the right to disqualify teams for misbehaviour or breaking the rules.

What do you need

- You are only allowed to use a Italian/English dictionary and a non scientific calculator. The use of hardware (including phones, tablet set,c.) is not approved, with exceptions of interface and discussing via Google Meet, watch this booklet or medical equipment.
- No books or other sources, except for this exercise booklet and a dictionary are to be consulted during the competition

What you are required to do

- The language used in the preliminary and international competition is English.
- Solutions and procedures must be included in the answer paper.
- All exercises must be handed in separately. Please use a separate sheet for each problem.
- Respect all the given instructions.
- Enjoy and have a great physics time!

Problems

1 Optical Interference

Prof. Marco Cannas - University of Palermo

A beam of polychromatic light ($300\,\mathrm{nm} \leq \lambda \leq 700\,\mathrm{nm}$) incises perpendicularly on a diffraction grating of length $h = 6\,\mathrm{cm}$ and composed of $N = 2 \times 10^4$ slit, each of which has a width $a = 1.5\,\mu m$.
After diffraction, the beam is collimated with a converging lens of focal distance $f = 60\,\mathrm{cm}$, into a screen S at a distance f from the lens.
Determine:

1. How many and which interference orders allow the observation of the entire spectrum; *(5 points)*

2. The linear dispersion at the first order of interference and the position in the screen where you can see the entire spectrum; *(5 points)*

3. The least difference $\Delta\lambda$ that can be solved at the extremes of the spectrum ($300\,\mathrm{nm} \leq \lambda \leq 700\,\mathrm{nm}$); *(5 points)*

4. Comment qualitatively what disadvantages you will encounter if the width of the slits is different from a. *(5 points)*

2 Electron band theory of solids

Prof. Francesco Ciccarello - University of Palermo & NEST

The (hypothetical) crystal in the figure features a large number of identical hydrogen-like atoms, whose equilibrium positions lie on the centers of black circles. Let $\phi(r)$ be the s-type ground-state orbital of a generic atom and $\widehat{\mathbf{H}}_e$ the electronic Hamiltonian (under the usual independent-electron approximation) when the nuclei are fixed to the aforementioned equilibrium positions. The following matrix elements are known:

$$\varepsilon = \int d\boldsymbol{r}\phi(\boldsymbol{r})\widehat{\mathbf{H}}_{\mathbf{e}}\phi(\boldsymbol{r}) = -14\,\mathrm{eV} \quad , \quad \gamma(b) = t_b = -1.5\,\mathrm{eV}$$

$$\gamma\Big(\frac{1}{2}\sqrt{a^2+b^2}\Big) = t_c = -2\,\mathrm{eV}$$

wiht

$$\gamma(R) = -\int d\boldsymbol{r}\phi(\boldsymbol{r})\widehat{\mathbf{H}}_{\mathbf{e}}\phi(\boldsymbol{r}-\boldsymbol{R})$$

Assuming that the only overlaps to take into account are those between orbitals whose distance is b and $\frac{1}{2}\sqrt{a^2+b^2}$ (thus all the remaining ones can be neglected), determine:

1. The primitive unit cell and the number of expected electron bands; *(2 points)*
2. The main symmetries of the lattice; *(6 points)*
3. The single-electron energy spectrum (under the usual independent-electron approximation); *(10 points)*
4. Whether the system is a conductor, an insulator or a semiconductor. *(2 points)*

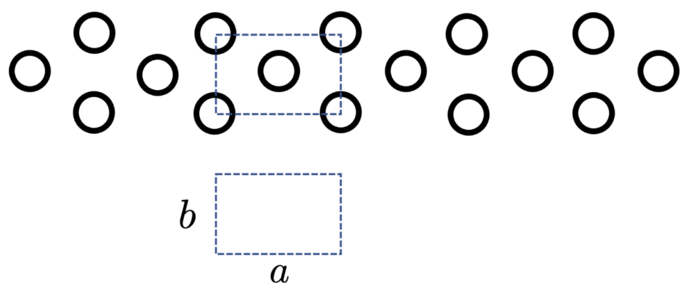


Figura 1

3 Thermodynamics: Compressibility charts

Prof. Matteo Pierno - University of Padova

The simplest reference to describe the thermodynamic properties of a gas is the approximation of the ideal or perfect gas, generalizing the historical and phenomenological gas laws of Boyle and Gay-Lussac. The level of precision with which the equation of state of a real fluid can be approximated to that of the ideal gas is described by a single comprehensive parameter called compressibility factor, defined by the ratio:

$$Z = \frac{pV_m}{RT} \tag{1}$$

p, V_m, T being the pressure, molar volume and temperature of the fluid, respectively, $R = 8.314 \, \frac{\mathrm{J}}{\mathrm{Kmol}}$ being the universal gas constant. Clearly, for an ideal gas, the function $Z(p, T)$ maintains the constant value $Z = 1$, while, for real gases, it depends on the state considered. The graphic representation of this function is equivalent to knowing the equation of state of a particular substance. In technical applications, this parameter is represented in a plane (Z, p) by a family of curves, one for each temperature. This representation is commonly known as compressibility chart. It can be prepared for any pure substance, or it can be calculated from a given equation of state like, for instance, the van der Waals equation of the state. In both cases, with respect to the law of corresponding states, the chart is represented in reduced variables, i.e., in units of the critical variables. In this case, we usually speak of generalized compressibility chart. In Figura 2 the generalized compressibility chart is obtained from the LeeKesler equation of state, an accurate equation of state based on 12 parameters. Let us consider a cylinder containing 5 kg of Carbon Dioxide (CO_2, Molecular Weight $M_{CO2} \simeq 44$) at the temperature $\vartheta_1 \simeq 122.5\,°\mathrm{C}$ and at the pressure $p_1 \simeq 221.4\mathrm{bar}$. The critical pressure and temperature of carbon dioxide are $p_c = 73.7\,\mathrm{bar}$ and $T_c = 304.36\,\mathrm{K}$ respectively.

1. Determine the volume of the cylinder and compare the result with the value that would be obtained by treating Carbon Dioxide as an ideal gas. *(15 points)*

2. The gas is then heated, at constant pressure, to the temperature $\vartheta_2 \simeq 183.4\,°\mathrm{C}$. Determine the amount of work done on the gas. *(5 points)*

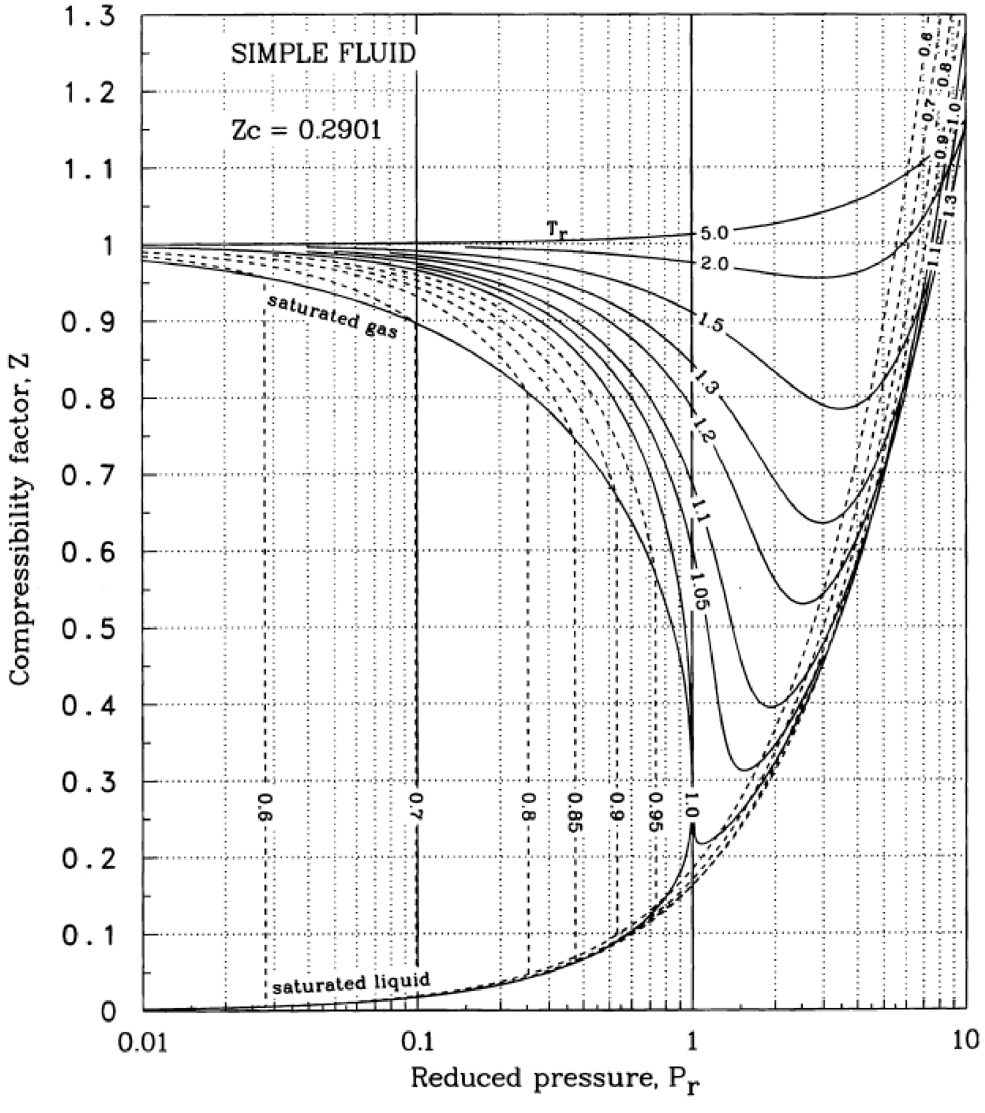


Figura 2: Generalized compressibility chart as a function of the reduced pressure, reproduced from Van Wylen et al., *Fundamentals of Classical Thermodynamics*.

4 Relativity: Bending of Light

Prof. Claudio Dappiaggi - University of Pavia

In 1801 Johann Georg von Soldner already predicted that light grazing the surface of the Sun would deflect. The following ideal model combines classical mechanics with special relativity to estimate the angle of deflection. Hence, consider a two dimensional Cartesian plane $\mathbb{R}^2$ and a Cartesian references frame endowed with the standard Euclidean coordinates. Thereon a photon is traveling at the speed of light along the x-axis and he is grazing the surface of the Sun. We assume that the Sun can be modeled as a disk of radius $R_\odot$ and of mass $M_\odot$ centered at $(0, y_0)$ with $y_0 < 0$, $|y_0| \leq R_\odot$ while that the gravitational potential still reads $V = -\frac{GM_\odot}{r}$.

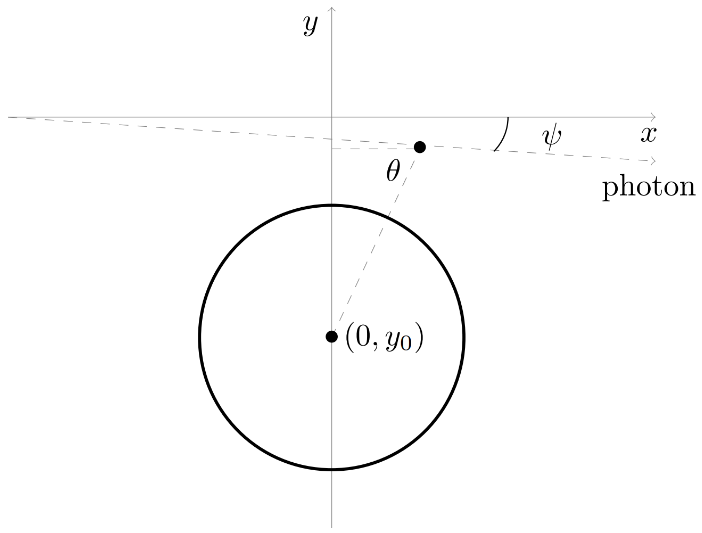


Figura 3

1. Compute ψ, the deflection angle of the photon due to the action of the Newtonian gravitational force assuming that the gravitational mass of the photon coincides with the relativistic one; *(15 points)*

2. Knowing that $M_\odot = 1.98 \times 10^{30}$Kg and $R_\odot \simeq 6.9 \times 10^8$m, evaluate ψ and compare the result with the prediction of the deflection angle from General Relativity, i.e. $\psi_{GR} = \frac{4GM_\odot}{R_\odot c^2}$ where G is the newton constant while c is the speed of light. *(5 points)*

Hint: When considering the vertical deflection of the photon, assume that the component of the velocity along the y-axis is very small in comparison to the one along the x-axis.

5 Quantum Mechanics

Prof. Giulio Pettini - University of Firenze

Suppose the Hamiltonian $\widehat{\mathbf{H}}$ for a particle of spin $\frac{1}{2}$ is

$$\widehat{\mathbf{H}} = \frac{\boldsymbol{p}^2}{2m} + \frac{2A}{\hbar}\boldsymbol{p}\cdot\boldsymbol{S} \tag{2}$$

1. Write the spectrum and the eigenfunction of $\widehat{\mathbf{H}}$; *(4 points)*
2. Calculate the time evolution operator $\widehat{\mathbf{U}}(t,0)$; *(4 points)*

At $t = 0$ the particle is in the state:

$$\Phi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{e^{\frac{i}{\hbar}\boldsymbol{P}\cdot\boldsymbol{x}}}{(2\pi\hbar)^{\frac{3}{2}}} \tag{3}$$

3. Find the spin-flip probability for the particle at generic time t; *(4 points)*

Modify the previous Hamiltonian by introducing a Coulomb potential:

$$\widehat{\mathbf{H}}_1 = \frac{\boldsymbol{p}^2}{2m} - \frac{e^2}{r} + \frac{2A}{\hbar}\boldsymbol{p}\cdot\boldsymbol{S} \tag{4}$$

Let's now consider the last term as a weak perturbation respect to the Coulomb interaction.

4. Has the spin degeneration of the ground state been removed? *(4 points)*
5. Find the conserved quantities of $\widehat{\mathbf{H}}_1$ and use them, with the parity symmetry, to discuss the perturbation matrix in $n = 2$. *(4 points)*