PrePlanks 2024

Associazione Italiana Studenti di Fisica

February 16th

Dear contestants, It is a pleasure to welcome you into the PLANCKS24 Italian Preliminaries! This booklet contains the exercises you will have to solve in order to access the PLANCKS24 Finals in Dublin, Ireland. You will face five different problems and are required to follow the instructions given in the next paragraphs.
Have fun and good luck!

Instruction

How it works

- 🔧 The contest consists of 5 exercises each worth 20 points. Subdivision of points are indicated in the exercises.
- 🔧 The test duration is for a total time of 3 hours.
- 🔧 When a problem is unclear, a participant can ask, via the crew, for a clarification from the organizing committee. The committee will respond to this request. If this response is relevant to all teams, we will provide this information to the other teams.
- 🔧 In situations to which no rule applies, the organization committee decides.
- 🔧 The organization has the right to disqualify teams for misbehaviour or breaking the rules.

What do you need

- 📜 You are only allowed to use a Italian/English dictionary and a non scientific calculator. The use of hardware (including phones, tablet set,c.) is not approved, with exceptions of an interface to discuss via Google Meet, a screen watch this booklet or medical equipment.
- 📜 No books or other sources, except for this exercise booklet and a dictionary are to be consulted during the competition.

What you are required to do

- 📜 The language used in the preliminary and international competition is English.
- 📜 Solutions and procedures must be included in the answer paper.
- 📜 All exercises must be handed in separately. Please use a separate sheet for each problem.
- 📜 Respect all the given instructions.
- 📜 Enjoy and have a great physics time!

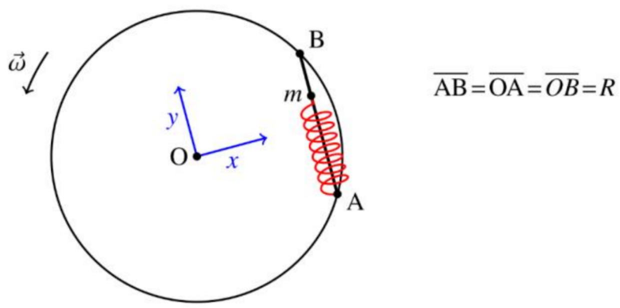
Problems

1 Classical Mechanics

Prof. Leonardo Banchi - University of Firenze

Consider the system shown in figure, composed of a disk with radius R and mass M, laying horizontal on a plane and constrained to rotate without friction around its center O, and a point mass m, constrained to move without friction along a guide AB (also of length R, so the triangle OAB is equilateral). The ends A and B of the guide are welded to the edges of the disk. A spring with elastic constant k and rest length $L_0 = \frac{2}{3}R$ connects the point mass m to point A (see the figure). Note that the acceleration due to gravity is orthogonal to the disk.

At first, an external motor keeps the disk rotating with constant angular velocity $\boldsymbol{\omega}$ (see the figure), with $\omega = \sqrt{\frac{2k}{m}}$.



1. Calculate the length of the spring at the equilibrium configuration (hint: perform the calculations in the Oxy mobile reference system shown in figure, where the y-axis is parallel to the guide AB at each instant). *(2 points)*

2. Calculate the minimum length of the spring, in the case where initially the point mass m starts from the end B with zero velocity relative to the guide. *(4 points)*

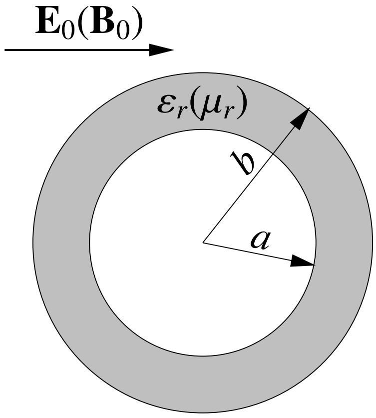
Now consider the case where the system is initially at rest with the point mass m at B, with the motor turned off, and consider its evolution.

3. Discuss whether in the subsequent motion the total momentum of the system, the total angular momentum of the system with respect to the pole O, or one of its components are conserved. *(7 points)*

4. Find the relationship between the angular velocity ω of the disk and the relative velocity $\dot{y}$ of the point mass, instant by instant. *(7 points)*

2 Electrodynamics in a shell

Prof. Andrea Macchi - University of Pisa



1. A spherical shell has inner radius a, outer radius $b > a$, and is made of a dielectric material of relative permittivity ϵ_r. The shell is located in a uniform external electric field $\mathbf{E}_0$. Calculate the electric field (assumed to be uniform) inside the shell. *(6 points)*

2. Consider the analogous problem of a shell of magnetic permeability μ_r located in an external magnetic field $\mathbf{B}_0$. Calculate the magnetic field inside the shell. *(3 points)*

3. A metallic shell of inner radius a and outer radius $b > a$ is placed in an uniform, oscillating electric field $\mathbf{E}(t) = \mathbf{E}_0{}^{-i\omega t}$. The frequency ω is supposed to be high enough that electrons are considered to be free without any effective friction force. Determine the frequencies for which the response of the shell is resonant. Consider in particular the limit of a thin shell having thickness $d = b - a \ll a$. *(6 points)*

4. In medical laser surgery, metallic nanoshells are inserted in organic tissue to locally increase the absorption of electromagnetic waves at frequencies for which the tissue is non-absorbing. Determine suitable fabrication parameters for a Gold nanoshell ($\omega_p \simeq 1.4 \times 10^{16}$ Hz) and a laser source operating at $\lambda_L = 800$ nm wavelength. *(5 points)*

3 Quantum Mechanics

Prof. Fabio Bagarello - University of Palermo

Let c and $H = H^\dagger$ be two operators satisfying the following conditions[1]:

$$cH = f(H)c \qquad , \text{and} \qquad [c, c^\dagger] = cc^\dagger - c^\dagger c = f(H) - H, \tag{1}$$

where $f(x)$ is a fixed function of x, strictly increasing. These are the defining properties of the so-called *generalized Heisenberg algebra* (GHA).

Let us assume that $e_0 \in \mathcal{H}$ is an eigenvector of H such that

$$He_0 = \epsilon_0 \, e_0, \tag{2}$$

with $\epsilon_0 > 0$.

Calling $e_n = (c^\dagger)^n e_0$, $n \geq 0$, the student should:

1. prove that the various e_n are eigenstates of H,

   $$He_n = \epsilon_n e_n,$$

   and deduce the expression of the eigenvalues ϵ_n, $n \geq 0$. *(4 points)*

2. prove that $\langle e_n, e_k \rangle = 0$ if $n \neq k$. *(2 points)*

3. prove that, if $\mathcal{F}_e = \{e_n, n \geq 0\}$ is complete in $\mathcal{H}$, then $ce_0 = 0$. *(2 points)*

4. prove that, again if $\mathcal{F}_e = \{e_n, n \geq 0\}$ is complete in $\mathcal{H}$,

   $$ce_n = (\epsilon_n - \epsilon_0)e_{n-1},$$

   $\forall n \geq 0$, with $\epsilon_{-1} := 0$, and

   $$c^\dagger c e_n = (\epsilon_n - \epsilon_0)e_n,$$

   $\forall n \geq 0$. *(4 points)*

5. prove that H can be rewritten as follows: $H = c^\dagger c + \epsilon_0 \mathbb{I}$. *(2 points)*

6. prove that, calling $H_{susy} = cc^\dagger + \epsilon_0 \mathbb{I}$, then $H_{susy} e_n = \epsilon_{n+1} e_n$. Prove also that $H_{susy} = f(H)$. *(3 points)*

7. show that an operator[2] a, satisfying $aa^\dagger - qa^\dagger a = \mathbb{I}$, $q \in]0, 1]$, produces an example of (1), for some suitable H and $f(x)$ to be identified by the student. *(3 points)*

[1] These formulas should be understood in the sense of unbounded operators, if needed. The student can neglect this technical aspect, and work with c and H as if they were bounded.

[2] Operators of this kind are called *quons* in the literature.

4 Thermodynamics: Walls Insulations

Prof. Matteo Pierno - University of Padova

A type of wall is composed of two layers of different thicknesses, which can be assumed to be two contacting slabs. The first slab, with thickness $s_1 = 0.05\,m$, is made of perforated brick with a thermal conductivity $\lambda_1 = 0.5\,\frac{W}{m\cdot K}$; the second slab, with thickness $s_2 = 0.10\,m$, is made of reinforced concrete, which has a thermal conductivity $\lambda_2 = 1.5\,\frac{W}{m\cdot K}$. The temperature on the left side of the first slab is $T_{P0} = 0°C$, while the temperature on the right side of the second slab is $T_{P2} = 20°C$.

1. Calculate the temperature T_{P1} of the surface separating the perforated brick from the reinforced concrete and the amount of heat exchanged per unit surface area and per unit time, neglecting the effects of convection and radiation. *(6 points)*

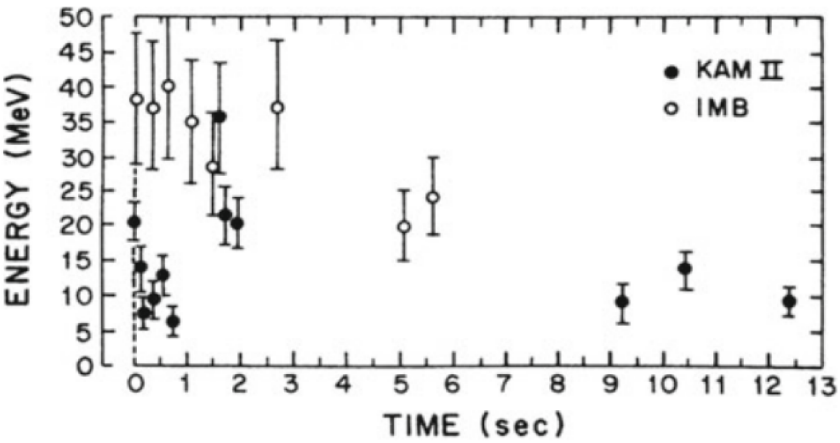
2. Assume that with this type of wall, two contiguous walls are built without openings, which delimit an interior space of a building, while two other walls, facing inward, separate the space from rooms kept at the same temperature. The two external walls have a length $l = 4$ m each and a height $h = 3$ m. The interior space is thermostatically controlled by a device that can operate as both an air conditioner and a heat pump according to a Carnot refrigeration cycle: in winter it transfers heat from the outside to the interior space, and in summer from the interior space to the outside. Calculate the coefficient of performance of the machine in the two configurations, knowing that $T_{P2} = 20$°C and that typically in winter $T_{P0} = 0$°C, while in summer $T_{P0} = 30$°C. *(5 points)*

3. Considering operation times of the machine in cooling and heating regimes equal to one hour each (approximated to an integer number of cycles), calculate the corresponding change in entropy of the Universe, including the walls in the environment. *(9 points)*

5 Particles: Space Neutrinos

Prof. Alessandro Menegolli - University of Pavia

On February 4$^{\text{th}}$ 1987, a number of neutrino detectors reported the observation of a neutrino burst, attributed to a Supernova event occurred in the Large Magellanic Cloud, at a distance $d = 4.8 \times 10^4$ pc from the Earth. Most of the events for which the energy and arrival time could be measured were due to electron antineutrino charged-current (CC) scattering $\overline{\nu}_e + p \rightarrow n + e^+$. The following figure is a scatter plot of energy and time for the Supernova $\overline{\nu}_e$ candidate events recorded by the water Cherenkov detectors Kamiokande-II (black dots) and IMB (white dots). Let's focus on the twelve Kamiokande-II events only and assume that a Supernova burst may last only a few seconds.



1. How do we obtain the antineutrino energy $E_{\overline{\nu}_e}$ from the positron energy E_{e^+}? *(2 points)*

2. Determine a lower bound for the electron neutrino lifetime. *(4 points)*

3. Using the Kamiokande-II data from the scatter plot, provide an upper bound to the electron neutrino mass. *(8 points)*

4. Estimate the average amount of light (i.e. the number of Cherenkov photons) collected by the Kamiokande-II photo-detection system after the interaction of a Supernova antineutrino inside the detector. *(6 points)*

Hints: the Kamiokande-II experiment consisted of a large water tank instrumented with photo-detectors sensitive to the visible light in the range $\lambda = 300 \div 500 nm$. *The refractive index of water for visible light can be approximated as a constant, with value* $n = 1.33$.