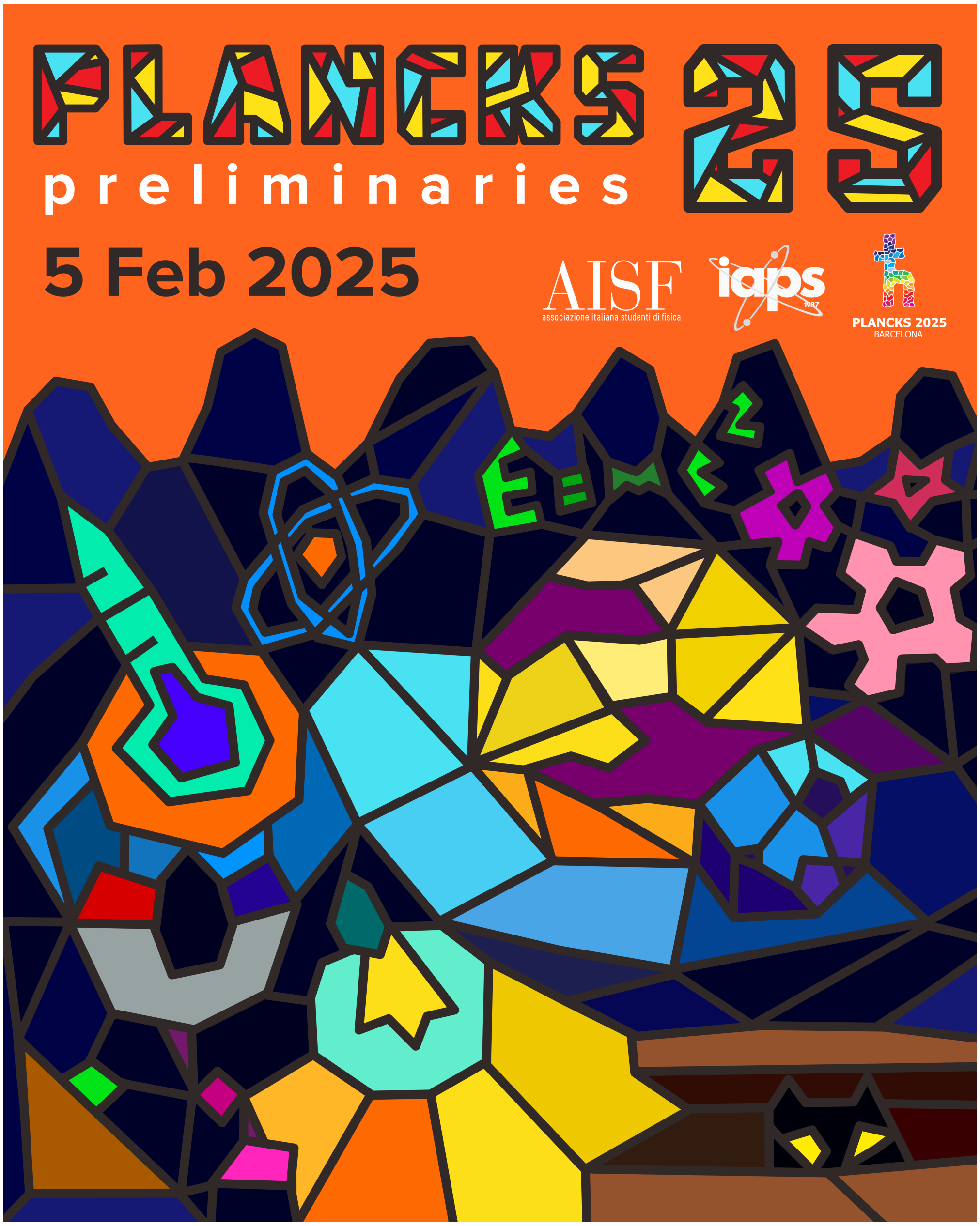
PrePlancks 2025

Associazione Italiana Studenti di Fisica

February 5th



Dear contestants, It is a pleasure to welcome you into the PLANCKS25 Italian Preliminaries! This booklet contains the exercises you will have to solve in order to access the PLANCKS25 Finals in Barcellona, Spain. You will face five different problems and are required to follow the instructions given in the next paragraphs.

Have fun and good luck!

Instruction

How it works

- The test consists of 5 exercises, each worth 20 points. Subdivision of points are indicated in the exercises.
- The test duration is for a total time of 3 hours.
- When a problem is unclear, a participant can ask, via the crew, for a clarification from the organizing committee. The committee will respond to this request. If this response is relevant to all teams, we will provide this information to the other teams.
- In situations to which no rule applies, the organization committee decides.
- The organization has the right to disqualify teams for misbehaviour or breaking the rules.

What do you need

- You are only allowed to use a Italian/English dictionary and a non scientific calculator. The use of hardware (including phones, tablet set, etc.) is not approved.
- No books or other sources, except for this exercise booklet and a dictionary, are to be consulted during the competition.

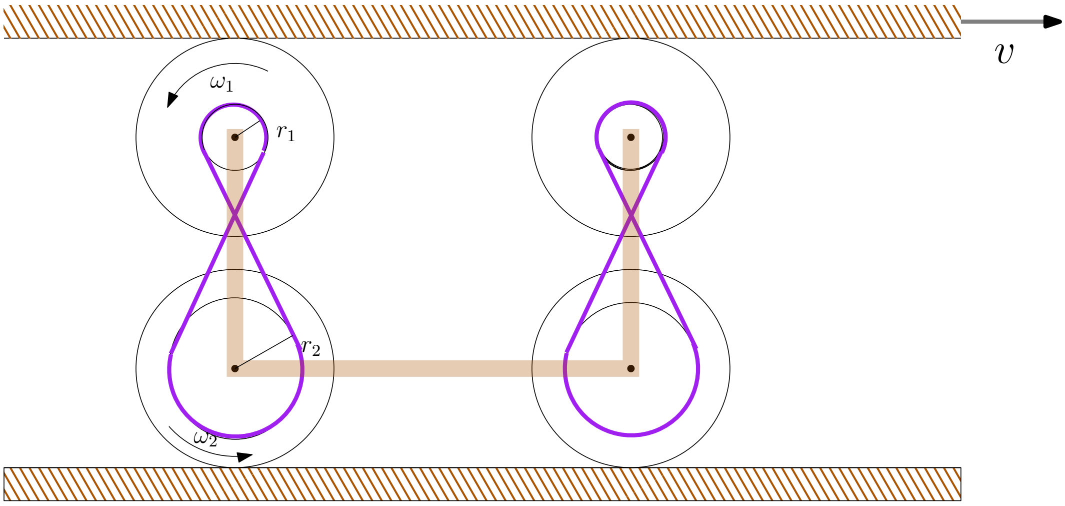
What you are required to do

- The language used in the preliminary and international competition is English.
- Solutions and procedures must be included in the answer paper.
- All exercises must be handed in separately. Please use a separate sheet for each problem and sign all the sheets.
- Respect all the given instructions.
- Enjoy and have a great physics time!

Problems

1 Classical Mechanics

Prof. Giancarlo Cella - University of Pisa



The cart represented in the figure has been assembled by connecting together four identical cylinders, with mass m and radius R, using a rigid structure with negligible mass. The cylinders rotate without slipping on the horizontal planes they are in contact with.

Each upper cylinder is connected with the lower one with a drive belt, adherent to two pulleys with radius r_1 and r_2. Friction is negligible.

The lower plane do not move, while the upper one moves horizontally with velocity $v > 0$.

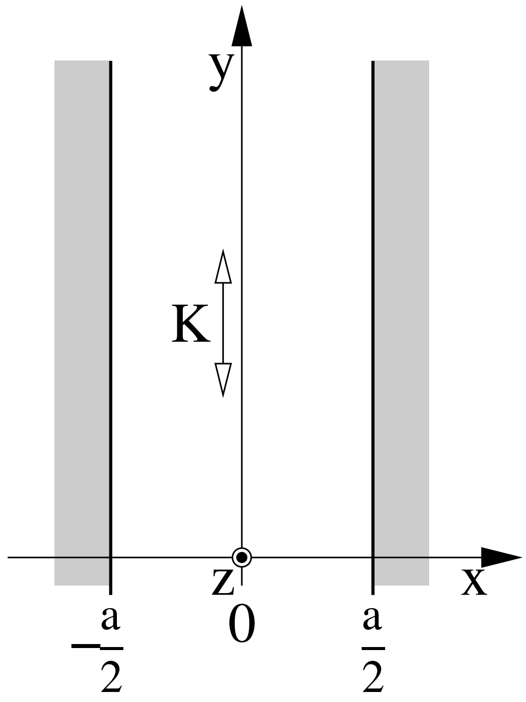
1. Evaluate the ratio $\gamma \equiv \omega_1/\omega_2$ between the angular velocities of upper and lower cylinders. *(5 points)*

2. Evaluate and plot the ratio between the velocity of the center of mass of the cart v_{cm} and the velocity of the upper plane, as a function of. Is it possible to have $v_{cm} = v$? Is it possible that $v_{cm} > v$? Is it possible that the cart moves in the opposite direction of the plane? What happens when $\gamma \to -1$? *(10 points)*

3. Suppose that the upper plane is not moving initially. Evaluate the work needed to change its velocity to v. Explain what happens when $\gamma \to -1$. *(5 points)*

2 Electrodynamics

Dott. Andrea Macchi - University of Pisa



An one-dimensional cavity model is composed by two conducting planes at $x = \pm a/2$ and an infinitely thin antenna at $x = 0$ with a current density $\mathbf{J}(x, y, z, t) = K(t)\,\delta(x)\,\hat{\mathbf{y}}$ with $K(t) = \mathrm{Re}[K_0\, e^{-i\omega t}]$, driven by an ideal current generator.

Assume in the first instance that the planes are made by perfect conductors.

1. Find the expression for the electromagnetic (EM) field and discuss the presence of resonances. *(6 points)*

2. Determine the instantaneous power exchanged between the EM field and the current generator. *(4 points)*

3. Now assume that the conductors have finite conductivity, so that they are characterized by a non-zero skin depth δ. Find again the EM field and the exchanged power (first discuss the case of resonant modes and then the general expression). *(10 points)*

3 Quantum Mechanics

Prof. Vittorio Lubicz - University of Roma Tre

Consider three particles of spin-$\frac{1}{2}$ in the state

$$|\psi_0\rangle = \frac{1}{\sqrt{3}} \left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle \right), \qquad (1)$$

where $|\uparrow\uparrow\downarrow\rangle$ is the eigenstate of the z-components S_{1z}, S_{2z}, S_{3z} of the spin of the three particles, corresponding to the eigenvalues $+\hbar/2$, $+\hbar/2$, $-\hbar/2$ respectively, and similarly for $|\uparrow\downarrow\uparrow\rangle$ and $|\downarrow\uparrow\uparrow\rangle$.

1. Find the possible results and corresponding probabilities of the measurement of S^2 and S_z, where $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2 + \boldsymbol{S}_3$ is the total spin of the three particles. *(5 points)*

2. Find the possible results and corresponding probabilities of the independent measurements of S_{1z}, $S_{1z} + S_{2z}$, and S_{1x} *(5 points)*

   Suppose that a measurement of the x-component S_{1x} of the spin of the first particle is carried out, leading to the result $S_{1x} = +\hbar/2$.

3. Find the state of the particles after the measurement and the possible results and corresponding probabilities of the independent measurements of S^2 and S_z. *(10 points)*

4 Statistical Mechanics

Prof. Enzo Orlandini - University of Padova

A one-dimensional chain is made by a sequence of N molecules having each a 2D ellipsoidal shape with minor and major axis of length b and a ($b < a$) respectively. Let us call x the direction of the 1D chain. Each molecule can be oriented in space in two directions, one in which a is oriented along x and the other in which b is oriented along x. Suppose that there is a force $\boldsymbol{F}$ applied to the N-esim molecule along the chain axis x. The energy of the system is given by the Hamiltonian

$$\mathcal{H} = -FL\left[\{l_i\}\right] = -F\sum_{i}^{N} l_i \tag{2}$$

where l_i can be either a or b.

1. Compute the canonical partition function of the system and its average energy $\langle E\rangle$. Verify the accuracy of the solution by calculating the limits of $\langle E\rangle$ at low and high temperatures. *(5 points)*

2. Compute the equilibrium average length $\langle L\rangle$ of the chain as a function of the force strength F and temperature T. Furnish the explicit expression of $\langle \ell\rangle \equiv \langle L\rangle/N$ as a function of F/T and obtain the limiting case of large forces ($Fb/k_BT >> 1$). *(5 points)*

3. Show that in the weak forces regime ($Fa/k_BT << 1$) the chain satisfies the relation

$$g(T)(\langle L\rangle - L_0) = F \tag{3}$$

   and determine $g(T)$ and L_0 as $L_0 = \ell_0 N$ *(5 points)*

4. Compute the free energy of the system, $f(T, F)$ as a function of temperature and $\langle L\rangle$ in the weak forces regime. *(5 points)*

5 QFT: A puzzling interacting theory

Prof. Angelo Esposito - University of Roma Sapienza

Consider a theory for a real massless scalar field, characterized by the free Lagrangian

$$\mathcal{L}_0 = \frac{1}{2}(\partial\phi)^2 \tag{4}$$

and by the following interaction term

$$\mathcal{L}_{\text{int}} = 2\lambda\phi(\partial\phi)^2 + 2\lambda^2\phi^2(\partial\phi)^2. \tag{5}$$

1. Compute the Feynman rules for the 3-point and 4-point interactions. *(7 points)*

2. Compute the on-shell connected amplitude for the $\phi\phi \to \phi\phi$ scattering. *(8 points)*

3. You should have found a peculiar result. Motivate it. *(5 points)*