PrePLANCKS 2026

Associazione Italiana Studenti di Fisica (AISF)

March 6th

Dear contestants, it is a pleasure to welcome you to the PLANCKS26 Italian Preliminaries!

This booklet contains the exercises you will need to solve to access the PLANCKS26 Finals in Eindhoven, Netherlands.
You will face five different problems and are required to follow the instructions given in the next paragraphs.

Have fun and good luck!

Instructions

How it works

- The test consists of 5 exercises, each worth 20 points. Subdivision of points is indicated in the exercises.
- The test duration is for a total time of 3 hours.
- When a problem is unclear, a participant can ask, via the crew, for clarification from the organising committee. The committee will respond to this request. If this response is relevant to all teams, we will provide this information to the other teams.
- In situations to which no rule applies, the organisation committee decides.
- The organisation has the right to disqualify teams for misbehaviour or rule violations.

What do you need

- You may use only an Italian/English dictionary and a non-graphic calculator. The use of hardware (including phones, tablet set, etc.) is not approved.
- No books or other sources, except for this exercise booklet and a dictionary, are to be consulted during the competition.

What you are required to do

- The language used in the preliminary and international competition is English.
- Solutions and procedures must be included in the answer paper.
- All exercises must be handed in separately. Please use a separate sheet for each problem and sign each sheet.
- Respect all the given instructions.

Enjoy and have a great physics time!

Problems

1 Classical Mechanics - Physics Basics of Curling

Prof. Ermanno Vercellin & Prof. Livio Bianchi - University of Turin

In the sport of curling a heavy stone is thrown on an ice field, and the combination of translational and rotational motion of the stone determines its trajectory on the ice. Let us simplify the matter. We model the stone as a homogeneous ring (zero width) with radius R and total mass m.

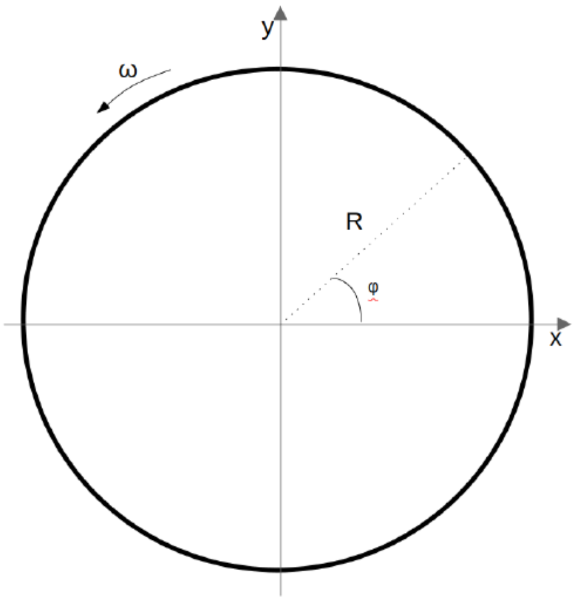

Figure 1: Curling stone as a homogeneous ring.

The stone is placed on the ice with its center of mass coinciding with the origin of a Cartesian axes system (x, y), and its angular velocity is ω (counter-clockwise). The center-of-mass velocity is initially zero. Figure 1 shows the situation.

The dynamic friction coefficient between the ice and the stone depends on the y-distance from the center of the stone as

$$\mu(y) = \mu_0 + \beta\,(y - y_{cm}),$$

where β is a positive constant.

Determine, at $t = 0$:

1. the friction force $\vec{F}_A$ acting on the stone; [6 points]
2. the torque $\vec{M}_A$ (moment of the force) generated by the friction force; [4 points]
3. the angular deceleration $\vec{\alpha}$. [4 points]

   Now imagine that the stone is in the same initial conditions as before (position and angular velocity), but with an initial velocity along the y-axis $\vec{v}_{cm}$.

4. In this case, calculate how the new friction force $\vec{F}'_A$ at $t = 0$ is different from $\vec{F}_A$, under the hypothesis that

$$|\vec{v}_{cm}| >> \omega R.$$

[6 points]

Congratulations! You now know (approximately) how to curl the stone and reach the goal. Now: exercise your sweeping and you'll be ready for the Olympics!

2 Classical Mechanics - Earth is (Almost) Round

Prof. Claudio Dappiaggi - University of Pavia

The surface of Earth can be approximated with very good precision by a 2-sphere S^2. At page 10 of volume 1 of the (British) Admiralty Manual of Navigation, it is written that:

> "The errors introduced by assuming a spherical Earth based on the international nautical mile are not more than 0.5% for latitude, 0.2% for longitude."

This exercise aims at validating this statement.
Hence, considering a sphere S^2 of radius $R > 0$:

1. Prove that the shortest path connecting two points thereon is a *great circle* (or *orthodrome*), namely the circular intersection between S^2 and a plane passing through its center point. [10 points]

2. Construct the *haversine formula* that computes the distance d between two points on the sphere as a function of their latitude and longitude. [8 points]

3. Setting the radius of Earth as 6378 km and assuming the distance between the cities of Oporto and New York to be 5465 km [1], estimate the error using the haversine formula, knowing that the coordinates of Oporto are 41.147658° latitude, −8.674770° longitude, while those of New York are 40.712776° latitude and −74.005974° longitude. [2 points]

Hint: Recall that, in absence of external forces, a point particle travelling between two points always follows the shortest path.

[1] Source: https://www.searates.com/distance-time/

3 Electromagnetism - The line and the cylinder

Prof. Giancarlo Maero - University of Milan

An infinite, straight line of charge with uniform linear charge density λ is placed at a distance $D > R_w$ from the axis of an infinitely long, perfectly conducting cylinder of radius R_w. The conductor is neutral and floating. We want to find the electrostatic potential in the domain represented by the whole space $\mathbb{R}^3$ minus the cylinder, using the method of image charges.

1. To do so, first choose a proper image charge distribution to solve the equivalent problem and reason about a suitable option for the magnitude of the image and the integration constants in the expression of the potential. [4 points]

2. Determine the position of the image charge distribution, coherently with the boundary conditions. [4.5 points]

3. Using the results obtained so far, rewrite the definitive expression of the potential, first at any point in the domain and then on the cylinder surface. Given $\lambda = 1\,\mathrm{nC/m}$, $R_w = 45\,\mathrm{mm}$ and $D = 2R_w$, calculate the numerical value of the potential on the cylinder. [2.5 points]

4. Determine the expression of the charge distribution induced by λ on the cylinder. Sketch a qualitative diagram of the distribution versus the position on the cylinder, and determine the position and value (both expression and numbers, using the data given in 3.) of its maxima and minima (absolute value). How do you expect the interaction force to be between the line charge and the cylinder? [4 points]

5. Based on the result obtained above, and without further calculations, deduce what happens if the line of charge λ is inside a cylindrical empty shell of conductor, i.e. $D < R_w$ (hence the problem's domain is $r < R_w$). What is the value and position of the image? [2 points]

6. Consider the latter case, i.e. $D < R_w$: A line of charge is placed inside a cylindrical conductive shell in vacuum. Add a uniform magnetic induction field $\vec{B} = B\hat{e}_z$, with $\hat{e}_z$ the cylinder's axis. It can be shown that the line of charge undergoes a rigid motion (as if it were a single body) with velocity $\vec{v} = \vec{E} \times \vec{B}/B^2$. Show that $\vec{v}$ has the form $\vec{v}(D) = v(D)\,\hat{e}_\theta$. Determine the full expression of $\vec{v}$ and of the angular rotation frequency ω. Considering $\lambda = 1\,\mathrm{nC/m}$, $B = 0.15\,\mathrm{T}$, $R_w = 45\,\mathrm{mm}$ and $D = R_w/2$, calculate the value of ω. [3 points]

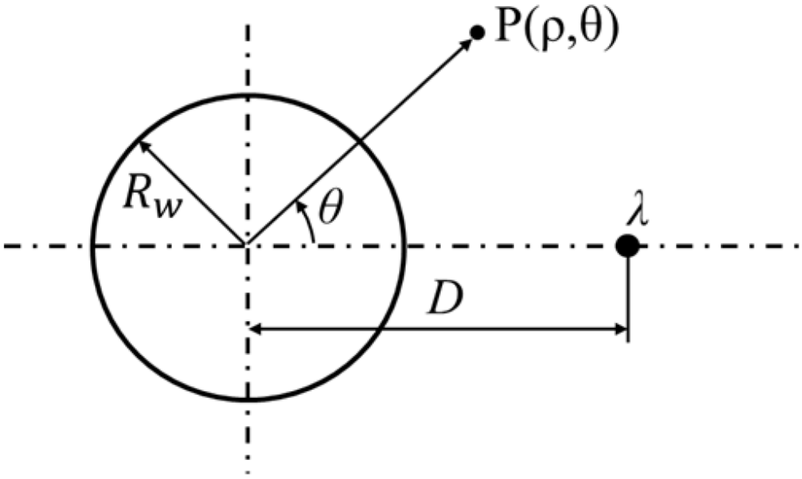

Figure 2: Sketch of the problem. The line of charge λ is placed in the vicinity of a cylindrical conductor.

4 Statistical Mechanics - 1D crystal in equilibrium at temperature T

Prof. Amos Maritan - University of Padova
Consider a one-dimensional array of $N+1$ particles of mass m aligned along the x-axis. The Hamiltonian of the system is:

$$H = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + u\big((x_i - x_{i-1})\big) \right]$$

where x_i is the position of the i-th particle, p_i is its conjugate momentum and

$$u(z) = \alpha z^4 \text{ if } z > 0$$

whereas $u(z) = +\infty$ if $z < 0$. The first particle is held fixed at the origin $x_0 = 0$.
The system is in thermal equilibrium at temperature T in contact with a heat bath (canonical ensemble). α is a positive constant.

1. Can we consider the particle indistinguishable? [2 points]
2. Determine the Helmholtz free energy F (hint: use the definition of Gamma function $\Gamma(a) = \int_0^\infty z^{a-1} e^{-z}\, dz$), [5 points]
3. the average energy $E = \langle H \rangle$, [3 points]
4. the entropy S [3 points]
5. the average length of the chain

$$L \equiv \left\langle \sum_{i=1}^{N} (x_i - x_{i-1}) \right\rangle$$

[3 points]

6. If in the above problem, we consider the potential

$$u(z) = \alpha z^4 \qquad \forall z \in \mathbb{R},$$

how would the average length of the chain be? Comment the answer calculating

$$L_2 = \sum_{i=1}^{N} \sqrt{\langle (x_i - x_{i-1})^2 \rangle}.$$

[6 points]

5 Quantum Mechanics - Sextic quantum Hamiltonian

Giulio Ticli - University of Trieste

Consider the following one-dimensional quantum Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar^2}{2m\lambda^2}\left(\frac{\hat{x}^6}{\lambda^6} - \frac{3\hat{x}^2}{\lambda^2}\right) \qquad \text{where } m > 0,\ \lambda > 0$$

1. Prove that $\hat{H}$ is positive-semi-definite. [7 points]
2. Prove that $\hat{H}$ has a non-degenerate spectrum in $L^2(\mathbb{R})$. [4 points]
3. Find the ground state $\psi_0(x)$, up to normalization. [7 points]
4. Normalize $\psi_0(x)$. [2 points]